# Shepard - A Fast Exact Match Short Read Aligner 

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## Overview

- The Challenge
- What is Shepard?
- Software for Minimal Perfect Hash Table Creation
- FPGA hardware pipeline for fast Hash Table lookups
- How fast is it?
- What's next?


## The Challenge

- Align millions of short DNA sequences to a reference genome.
- Current aligners (simplified):

- Our Solution:
- One giant hash table


## What is a MPH?

- MPH = Minimal Perfect Hash
- Given a fixed set of keys...
- Perfect Hashes have no collisions
- Minimal Perfect Hashes contain no empty buckets (they are memory efficient)
- For large sets of keys, you can create a general MPH in O(n) time.
- General MPH algorithms require using an intermediate table.
- The intermediate table is the MPH.


## Minimal Perfect Hash Function Example

- Hash the key once to retrieve a few bits of information stored in the intermediate table
- Do one of the following:
- Rehash with the new seed from the intermediate table (any buckets that had a collision would be rehashed*)
- Add the offset from the intermediate table to the initial index.
- The MPH is created by choosing values for the intermediate table so that a given set of keys will not collide!


## Minimal Perfect Hash Example



## Shepard's Software

- Differences between our algorithm and other general purpose MPH algorithms (CHD)
- CHD compresses the intermediate table down to between 2 and 4 bits per entry
- Shepard does not compress (no need); if entries are 8 byte aligned, there are 20 bits per entry available for the intermediate table



## Speed of Shepard's Hash Table Construction

- Current MPH creation using C code (CHD)
- 800,000 entries / sec

*only 20 million entries

- Shepard MPH C code
- 300,000 entries / sec *using 2.8 billion entries
- For 2.8 billion entries, takes ~2.5 hours

Number of Keys x Generation Time

(d) $\alpha=1.00$, Space lower bound $=1.44$.

Graph from: Hash, displace, and compress
Djamal Belazzougui et al

## Future Work

- Hardware Pipeline for Hash Table Creation
- The 6 stages for hash table creation are simple for-loops
- Assuming:
- ~40 memory operations per entry in construction
- Application is memory bound
- Convey HC-1 can perform 10 billion memory ops / sec
- We could create the hash table in about 12 seconds, more than 250x faster than CHD
- Concurrency Issues


## Software Implementation

```
Algorithm 1 Pseudocode of the Shepard Pipeline
    procedure ALIGN(reads, genome, hashtable, results)
        for \((i=0 ; i<\) length \((\) reads \() ; i++\) ) do
            \(r \leftarrow \operatorname{reads}[i] ; \quad \triangleright\) Stage 1
            \(h \leftarrow \operatorname{hash}(r\), seed \(=0) ; \quad \triangleright\) Stage 2
            ivalue \(\leftarrow\) intermediateTable \([h]\);
            if (ivalue is an offset) then
                \(\triangleright\) Stage 3
                index \(\leftarrow \operatorname{hash}(r\), seed \(=0)+\) ivalue;
            else
                index \(\leftarrow \operatorname{hash}(r\), seed \(=\) ivalue \() ;\)
            end if
            entry \(\leftarrow\) hashtable[index];
            check \(\leftarrow\) genome [entry.index];
                    \(\triangleright\) Stage 4
            if ( \(r==\) check) then
                results \([i] \leftarrow\) entry;
            else
                results \([i] \leftarrow N U L L ;\)
            end if
            end for
    end procedure
```



## Software Implementation

- Speed
- Single threaded:
- 1,000,000 reads / second (using total program execution time)
- Performance increases when using multiple threads

PERFORMANCE COMPARISON OF POPULAR SHORT READ ALIGNERS.

| Tool | Platform | Speed (reads/s) | \% Aligned | Memory (GB) |
| :--- | :--- | ---: | ---: | ---: |
| MAQ [4] | CPU | 50 | 93.2 | 1.2 |
| SOAP | CPU | 70 | 93.8 | 14.7 |
| SOAP2 [5] | CPU | 2,000 | 93.6 | 5.4 |
| Bowtie [6] | CPU | 2,500 | 91.7 | 2.3 |
| SOAP3 [9] | GPU | $6,000,000$ | 96.8 | 3.2 |

## The Convey HC-1



## The Convey HC-1



## Convey Custom AE Development Process

- It's simple!
- Take your original program, pull out a kernel written in C/C++
- Turn the kernel into a CAE instruction (define ISA)
- Write a software simulator of the program
- Emulates custom hardware
- Validates memory accesses
- Validates AEG registers
- Write hardware description in Verilog or VHDL
- Use the software simulator to test your design (quick)
- Build a bitfile (slow)


## Hardware Implementation



## Hash Function Pipeline

- Jenkin’s Spooky Hash
- Fixed the pipeline specifically for 100 base pair reads (25 bytes)
- Hash function consists of rotations, additions, and XOR


## Hash Function Pipeline Implementation

First 16 Bytes - Mix Pipeline


Last 9 Bytes - Ending Mix Pipeline


## Hash Function Pipeline Implementation

Mix Block


Last 9 Bytes - Ending Mix Block


## Speed

- 350,000,000 reads per second

PERFORMANCE COMPARISON OF POPULAR SHORT READ ALIGNERS.

| Tool | Platform | Speed (reads/s) | \% Aligned | Memory (GB) |
| :--- | :--- | ---: | ---: | ---: |
| MAQ [4] | CPU | 50 | 93.2 | 1.2 |
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| Bowtie [6] | CPU | 2,500 | 91.7 | 2.3 |
| SOAP3 [9] | GPU | $6,000,000$ | 96.8 | 3.2 |
| Shepard | FPGA | $350,000,000$ | 25.2 | 23.3 |

- Exact matches only!
- \% aligned depends on the quality of the reference genome and the read data
- Had we used better read data (such as the data used in the SOAP3 paper), the \% aligned would be as high as 60.3\%


## Future Work

- Let's update the pipeline to allow mismatches!
- The idea:
- Split the read into n parts. Use a hash table to lookup the index of these parts.
- Compare the indices of the parts. If any of the indices match, we can compare the read to the genome to see how many mismatches occurred!
- The time cost lies in the extra $2 * \mathrm{n}$ memory operations that must be performed.
- Our original design only used 12 of the 16 memory controllers, so if $\mathrm{n}=3$, we would incur no time penalty!
- We get a "free lunch" by using all of the available resources.


## Future Work



## Future Work Example

AAAAAAAAAACCCCGCCCCCTTTTTTTTTT (Read length = 30)


## Future Work Example



Let's pull 30 base pairs from the reference genome at index 1000.

## AAAAAAAAAACCCCCCCCCCTTTTTTTTTT

Index 1000 matched more than one part of the read!

## AAAAAAAAAACCCCGCCCCCTTTTTTTTTTT

Compare Read with Genome

## Future Work Example

- The comparison of the genome and a read is fairly simple in hardware for mismatches:
- XOR the genome with the read
- Count the number of 1's. If all 0's, it's an exact match.
- It becomes slightly more complex for insertions/deletions, but the same approach can be taken for comparison


## Future Work Risks

- What affect will the presence of duplicates have on the alignment \%?
- At 100 base pairs, 2.36\% of the genome is a duplicate of itself.
- At 36 base pairs, 11\% of the genome is a duplicate of itself.
- Let's test in software and find out!


## Conclusion (1/2)

- We implemented our own software to create MPH table:
- Our Speed:
- Other software (CHD):

300,000 800,000 entries / sec

- Future Work
- Increase the speed to 250,000,000 entries / sec
- This would allow us to make the pre-processing step part of the alignment process!
- Instead of using a generic human reference genome, people may be able to use the DNA sequence of a blood family member as a reference in order to increase the percentage of exact matches.
- The multi-port cache is useful IP for future projects utilizing the Convey HC-1 (allows atomic read-write)


## Conclusion (2/2)

- We implemented our own hardware for read alignment:
- Speed: 350,000,000 reads / sec
- Alignment: 25\%
- About 60x speedup over SOAP3 (GPU)
- Over 100,000x speedup over Bowtie (CPU)
- Con: exact match only
- Future Work
- Increase the alignment percentage by gaining the ability to detect mismatches, insertions, and deletions using the hashing approach.
- This would make the project applicable to real-world sequence alignment. We can alter the pipeline without loosing too much speed.


## Questions

## Future Work

## Future Work - Concurrency Solution

- Divide the hash table and keys in four, one for each of the four FPGAs (AEs) on the Convey HC-1
- Use the first two bits of the key to separate the hash table
- Need to keep track of the size of each hash table, though they should be roughly the same size
- This gets rid of AE to AE concurrency issues by separating the problem
- For stages 2,4 , and 5 , implement a cache on each AE for handling the rare case of a concurrency issues.


## Future Work - The Cache



## Future Work - The Cache

- 16 Cache ports
- Use the Write Complete interface
- No crossbar, no read order queue, no strong order queue
- Write back on replace
- The cache would need the ability to Lock certain cache lines to a given Cache Port. Store the lock with the cache line.
- Lock Order Queue
- If a request for a memory address comes in, and the cache is locked, the request waits until it is unlocked
- The cache will provide an infrastructure that we can use more generally in other projects utilizing the Convey HC-1. It allows strongly-ordered memory operations across multiple Memory Controllers.


## Construction Algorithm Visualized!

## Algorithm

- Stage 1 (create unique)
- Create a list of all $N$ of your <key, value> pairs to be added to the hash table
- Stage 2
- Hash every key into a bucket to see which spaces in the hash table will have collisions
- This amounts to having an array of size N initialized to all zeros. Then, you hash every key and increment the value at that index.
- Stage 2a
- Count the number of buckets of each size. Since the max bucket size is small (<20), this can be done during Stage 2 .


## Algorithm (Continued)

- Stage 3 (sorting)
- Sort the keys by bucket size (largest to smallest)
- Using the collision count values from stage 2 and the size of each bucket, this can be done in $\mathrm{O}(\mathrm{N})$ time.
- Stage 4 (reseed big buckets)
- For each bucket size >= 2 , starting with the largest, reseed the bucket
- This involves initializing an bit array with all 0's.
- For each bucket (contains 2 or more keys), rehash all the keys from the bucket with a new seed. Check the bit array that all keys have a spot in the final array
- Store the new seed in the intermediate table.


## Algorithm (Continued)

- Stage 5 (displace singular buckets)
- For each bucket size = 1, find the next available empty space in the bit array
- Place the key in this position, recording the offset from the original location in the intermediate table
- Stage 6 (add values to hash table)
- The intermediate table is your a MPHF
- Simply place the values in the final output table.


## Example

- We are going to create a MPH Table for the following keys:
- Armadillo
- Bird
- Cat
- Dog
- Elephant
- Frog
- Garage
- Horse
- Iguana
- Jaguar


## Stage 2

- Keys:
- Hash(seed=0)

| Bucket Size | Number |
| :---: | :---: |
| 0 | 10 |
| 1 | 0 |
| 2 | 0 |


| Counts |
| :---: |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |

## Stage 2

- Keys:
- Hash(seed=0)
- Armadillo - 1

| Bucket Size | Number |
| :---: | :---: |
| 0 | 9 |
| 1 | 1 |
| 2 | 0 |


| Counts |
| :---: |
| 0 |
| 1 |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |

## Stage 2

- Keys:
- Hash(seed=0)
- Armadillo - 1
- Bird - 5

| Bucket Size | Number |
| :---: | :---: |
| 0 | 8 |
| 1 | 2 |
| 2 | 0 |


| Counts |
| :---: |
| 0 |
| 1 |
| 0 |
| 0 |
| 0 |
| 1 |
| 0 |
| 0 |
| 0 |
| 0 |

## Stage 2

- Keys:
- Hash(seed=0)
- Armadillo - 1
- Bird - 5
- Cat
- 4

| Bucket Size | Number |
| :---: | :---: |
| 0 | 7 |
| 1 | 3 |
| 2 | 0 |


| Counts |
| :---: |
| 0 |
| 1 |
| 0 |
| 0 |
| 1 |
| 1 |
| 0 |
| 0 |
| 0 |
| 0 |

## Stage 2

- Keys:
- Armadillo - 1
- Bird - 5
- Cat
- 4
- Dog -8

| Bucket Size | Number |
| :---: | :---: |
| 0 | 6 |
| 1 | 4 |
| 2 | 0 |


| Counts |
| :---: |
| 0 |
| 1 |
| 0 |
| 0 |
| 1 |
| 1 |
| 0 |
| 0 |
| 1 |
| 0 |

## Stage 2

- Keys:
- Armadillo - 1
- Bird - 5
- Cat
- 4
- Dog
- Elephant
$-3$

| Bucket Size | Number |
| :---: | :---: |
| 0 | 5 |
| 1 | 5 |
| 2 | 0 |

## Stage 2

- Keys:
- Armadillo - 1
- Bird - 5
- Cat
- Dog
- Elephant
- 3
- Frog - 1

| Bucket Size | Number |
| :---: | :---: |
| 0 | 5 |
| 1 | 4 |
| 2 | 1 |


| Counts |
| :---: |
| 0 |
| 2 |
| 0 |
| 1 |
| 1 |
| 1 |
| 0 |
| 0 |
| 1 |
| 0 |

## Stage 2

- Keys:
- Armadillo - 1
- Bird - 5
- Cat
- Dog
- Elephant
- 3
- Frog - 1
- Garage - 9

| Bucket Size | Number |
| :---: | :---: |
| 0 | 4 |
| 1 | 5 |
| 2 | 1 |


| Counts |
| :---: |
| 0 |
| 2 |
| 0 |
| 1 |
| 1 |
| 1 |
| 0 |
| 0 |
| 1 |
| 1 |

## Stage 2

- Keys:
- Armadillo - 1
- Bird - 5
- Cat
- Dog
- Elephant
- Frog
- Garage
- Horse - 5

| Bucket Size | Number |
| :---: | :---: |
| 0 | 4 |
| 1 | 4 |
| 2 | 2 |

- 3
- 1
- 9

Counts

- 8


## Stage 2

- Keys:
- Armadillo - 1
- Bird - 5
- Cat
- Dog
- Elephant
- Frog
- Garage
- Horse - 5
- Iguana - 0

| Bucket Size | Number |
| :---: | :---: |
| 0 | 3 |
| 1 | 5 |
| 2 | 2 |

$-3$

- 1
- 9

Counts

- 8


| Counts |
| :---: |
| 1 |
| 2 |
| 0 |
| 1 |
| 1 |
| 2 |
| 0 |
| 0 |
| 1 |
| 1 |

## Stage 2

- Keys:
- Armadillo - 1
- Bird - 5
- Cat
- Dog
- Elephant
- Frog
- Garage
- Horse
- Iguana
- Jaguar
- Hash(seed=0)
- 4
$-8$
- 3
- 1
- 9
- 5
- 0
$-7$

| Bucket Size | Number |
| :---: | :---: |
| 0 | 2 |
| 1 | 6 |
| 2 | 2 |


| 1 |
| :--- |
| 2 |
| 0 |
| 1 |
| 1 |
| 1 |

## Stage 2

- Keys:
- Armadillo - 1
- Bird - 5
- Cat
- Dog
- Elephant
- Frog
- Garage
- Horse
- Iguana
- Jaguar
- Hash(seed=0)
- 4
$-8$
- 3
- 1
- 9
- 5
- 0
$-7$

| Bucket Size | Number |
| :---: | :---: |
| 0 | 2 |
| 1 | 6 |
| 2 | 2 |


| 1 |
| :--- |
| 2 |
| 0 |
| 1 |
| 1 |
| 1 |

## Stage 3

$$
\begin{aligned}
& \text { Keys: } \\
& \text { - Armadillo } \\
& \text { - Bird } \\
& \text { - Cat } \\
& \text { - Dog } \\
& \text { - Elephant } \\
& \text { - Frog } \\
& \text { - Garage } \\
& \text { - Horse } \\
& \text { - Iguana } \\
& \text { - Jaguar }
\end{aligned}
$$

| Bucket Size | Number |
| :---: | :---: |
| 0 | 2 |
| 1 | 6 |
| 2 | 2 |
|  |  |
| New Index | Counts |
| - | 1 |
| - | 2 |
| - | 0 |
| - | 1 |
| - | 1 |
| - | 2 |
| - | 0 |
| - | 1 |
| - | 1 |
| - | 1 |

## Stage 3

- Keys:
- Armadillo
- Bird
- Cat
- Dog
- Elephant
- Frog
- Garage
- Horse
- Iguana
- Jaguar


| Bucket Size | Number |
| :---: | :---: |
| 0 | 2 |
| 1 | 6 |
| 2 | 2 |
|  |  |
| New Index | Counts |
| 4 | 1 |
| - | 2 |
| - | 0 |
| - | 1 |
| - | 1 |
| - | 2 |
| - | 0 |
| - | 1 |
| - | 1 |
| - | 1 |

## Stage 3

- Keys:
- Armadillo
- Bird
- Cat
- Dog
- Elephant
- Frog
- Garage
- Horse
- Iguana
- Jaguar


| Bucket Size | Number |
| :---: | :---: |
| 0 | 2 |
| 1 | 6 |
| 2 | 2 |
|  |  |
| New Index | Counts |
| 4 | 1 |
| 0 | 2 |
| - | 0 |
| - | 1 |
| - | 1 |
| - | 2 |
| - | 0 |
| - | 1 |
| - | 1 |
| - | 1 |

## Stage 3

- Keys:
- Armadillo
- Bird
- Cat
- Dog
- Elephant
- Frog
- Garage
- Horse
- Iguana
- Jaguar


| Bucket Size | Number |
| :---: | :---: |
| 0 | 2 |
| 1 | 6 |
| 2 | 2 |
|  |  |
| New Index | Counts |
| 4 | 1 |
| 0 | 2 |
| - | 0 |
| 5 | 1 |
| - | 1 |
| - | 2 |
| - | 0 |
| - | 1 |
| - | 1 |
| - | 1 |

## Stage 3

- Keys:
- Armadillo
- Bird
- Cat
- Dog
- Elephant
- Frog
- Garage
- Horse
- Iguana
- Jaguar


| Bucket Size | Number |
| :---: | :---: |
| 0 | 2 |
| 1 | 6 |
| 2 | 2 |
|  |  |
| New Index | Counts |
| 4 | 1 |
| 0 | 2 |
| - | 0 |
| 5 | 1 |
| 6 | 1 |
| - | 2 |
| - | 0 |
| - | 1 |
| - | 1 |
| - | 1 |

## Stage 3

- Keys:
- Armadillo
- Bird
- Cat
- Dog
- Elephant
- Frog
- Garage
- Horse
- Iguana
- Jaguar


| Bucket Size | Number |
| :---: | :---: |
| 0 | 2 |
| 1 | 6 |
| 2 | 2 |
|  |  |
| New Index | Counts |
| 4 | 1 |
| 0 | 2 |
| - | 0 |
| 5 | 1 |
| 6 | 1 |
| 2 | 2 |
| - | 0 |
| - | 1 |
| - | 1 |
| - | 1 |

## Stage 3

- Keys:
- Armadillo
- Bird
- Cat
- Dog
- Elephant
- Frog
- Garage
- Horse
- Iguana
- Jaguar


| Bucket Size | Number |
| :---: | :---: |
| 0 | 2 |
| 1 | 6 |
| 2 | 2 |
|  |  |
| New Index | Counts |
| 4 | 1 |
| 0 | 2 |
| - | 0 |
| 5 | 1 |
| 6 | 1 |
| 2 | 2 |
| - | 0 |
| 7 | 1 |
| - | 1 |
| - | 1 |

## Stage 3

- Keys:
- Armadillo
- Bird
- Cat
- Dog
- Elephant
- Frog
- Garage
- Horse
- Iguana
- Jaguar


| Bucket Size | Number |
| :---: | :---: |
| 0 | 2 |
| 1 | 6 |
| 2 | 2 |
|  |  |
| New Index | Counts |
| 4 | 1 |
| 0 | 2 |
| - | 0 |
| 5 | 1 |
| 6 | 1 |
| 2 | 2 |
| - | 0 |
| 7 | 1 |
| 8 | 1 |
| - | 1 |

## Stage 3

- Keys:
- Armadillo
- Bird
- Cat
- Dog
- Elephant
- Frog
- Garage
- Horse
- Iguana
- Jaguar


| Bucket Size | Number |
| :---: | :---: |
| 0 | 2 |
| 1 | 6 |
| 2 | 2 |
|  |  |
| New Index | Counts |
| 4 | 1 |
| 0 | 2 |
| - | 0 |
| 5 | 1 |
| 6 | 1 |
| 2 | 2 |
| - | 0 |
| 7 | 1 |
| 8 | 1 |
| 9 | 1 |

## Stage 3

- Keys:
- 1- Armadillo
-5- Bird
- 4- Cat
- 8- Dog
- 3- Elephant
- 1- Frog
- 9- Garage
- 5- Horse
- 0- Iguana
- 7- Jaguar
- Sorted Keys:

| - Armadillo | 0 | 2 |
| :--- | :---: | :---: |
| - Frog | 1 | 6 |
| - Bird | 2 | 2 |
| - Horse |  |  |
| - Igw Index | Counts |  |
|  | 0 | 1 |
| - Elephant | - | 2 |
| - Cat | 5 | 0 |
| - Jaguar | 2 | 1 |
| - Dog | - | 1 |
| - Garage | 7 | 2 |
|  | 9 | 1 |


| Bucket Size | Number |
| :---: | :---: |
| 0 | 2 |
| 1 | 6 |
| 2 | 2 |

New Index
4

## Stage 4

- From largest to smallest
- Armadillo
- 1 -> 2
- Frog
- 1 -> 7
- Bird
$-5$
- Horse
- 5
- Iguana - 0
- Elephant
$-3$
- Cat
- 4
- Jaguar
$-7$
- Dog
$-8$
- Garage
$-9$

| Intermediate <br> Table | Bit Array |
| :---: | :---: |
| 0 | 0 |
| 0 | 0 |
| 0 | 0 |
| 0 | 0 |
| 0 | 0 |
| 0 | 0 |
| 0 | 0 |
| 0 | 0 |
| 0 | 0 |
| 0 | 0 |

## Stage 4

- From largest to smallest
- Armadillo - 1 -> 2
- Frog - 1 -> 7
- Bird - 5
- Horse - 5
- Iguana - 0
- Elephant - 3
- Cat
- 4
- Jaguar
$-7$
- Dog
$-8$
- Garage
$-9$

| Intermediate <br> Table | Bit Array |
| :---: | :---: |
| 0 | 0 |
| reseed = 1 | 0 |
| 0 | 1 |
| 0 | 0 |
| 0 | 0 |
| 0 | 0 |
| 0 | 0 |
| 0 | 1 |
| 0 | 0 |
| 0 | 0 |

## Stage 4

- From largest to smallest
- Armadillo - 1 -> 2
- Frog
- 1 -> 7
- Bird
- 5 -> 1
- Horse - 5 -> 9
- Iguana - 0
- Elephant
$-3$
- Cat
$-4$
- Jaguar
$-7$
- Dog
$-8$
- Garage
$-9$

| Intermediate <br> Table | Bit Array |
| :---: | :---: |
| 0 | 0 |
| reseed = 1 | 1 |
| 0 | 1 |
| 0 | 0 |
| 0 | 0 |
| reseed = 1 | 0 |
| 0 | 0 |
| 0 | 1 |
| 0 | 0 |
| 0 | 1 |

## Stage 5

- From largest to smallest
- Armadillo - 1 -> 2
- Frog
- 1 -> 7
- Bird
- 5 -> 1
- Horse
- 5 -> 9
- Iguana - 0 -> 0
- Elephant
$-3$
- Cat
- 4
- Jaguar
$-7$
- Dog
$-8$
- Garage
$-9$

| Intermediate <br> Table | Bit Array |
| :---: | :---: |
| offset = 0 | 1 |
| reseed = 1 | 1 |
| 0 | 1 |
| 0 | 0 |
| 0 | 0 |
| reseed = 1 | 0 |
| 0 | 0 |
| 0 | 1 |
| 0 | 0 |
| 0 | 1 |

## Stage 5

- From largest to smallest
- Armadillo - 1 -> 2
- Frog - 1 -> 7
- Bird - 5 -> 1
- Horse - 5 -> 9
- Iguana - $0->0$
- Elephant - 3 -> 0
- Cat
- 4
- Jaguar
$-7$
- Dog
$-8$
- Garage
$-9$

| Intermediate <br> Table | Bit Array |
| :---: | :---: |
| offset = 0 | 1 |
| reseed = 1 | 1 |
| 0 | 1 |
| offset = 0 | 1 |
| 0 | 0 |
| reseed = 1 | 0 |
| 0 | 0 |
| 0 | 1 |
| 0 | 0 |
| 0 | 1 |

## Stage 5

- From largest to smallest
- Armadillo - 1 -> 2
- Frog - 1 -> 7
- Bird
- 5 -> 1
- Horse - 5 -> 9
- Iguana - $0->0$
- Elephant - $3->0$
- Cat - 4 -> 0
- Jaguar - 7
- Dog
$-8$
- Garage
$-9$

| Intermediate <br> Table | Bit Array |
| :---: | :---: |
| offset = 0 | 1 |
| reseed = 1 | 1 |
| 0 | 1 |
| offset = 0 | 1 |
| offset = 0 | 1 |
| reseed = 1 | 0 |
| 0 | 0 |
| 0 | 1 |
| 0 | 0 |
| 0 | 1 |

## Stage 5

- From largest to smallest
- Armadillo - 1 -> 2
- Frog - 1 -> 7
- Bird
- 5 -> 1
- Horse - 5 -> 9
- Iguana - $0->0$
- Elephant - $3->0$
- Cat
- 4 -> 0
- Jaguar
- 7 -> 5
- Dog
$-8$
- Garage
$-9$

| Intermediate <br> Table | Bit Array |
| :---: | :---: |
| offset = 0 | 1 |
| reseed = 1 | 1 |
| 0 | 1 |
| offset = 0 | 1 |
| offset = 0 | 1 |
| reseed = 1 | 1 |
| 0 | 0 |
| offset =-2 | 1 |
| 0 | 0 |
| 0 | 1 |

## Stage 5

- From largest to smallest
- Armadillo - 1 -> 2
- Frog - 1 -> 7
- Bird
- 5 -> 1
- Horse - 5 -> 9
- Iguana - $0->0$
- Elephant - $3->0$
- Cat - $4->0$
- Jaguar
- 7 -> 5
- Dog
- 8 -> 6
- Garage
$-9$

| Intermediate <br> Table | Bit Array |
| :---: | :---: |
| offset = 0 | 1 |
| reseed = 1 | 1 |
| 0 | 1 |
| offset = 0 | 1 |
| offset = 0 | 1 |
| reseed = 1 | 1 |
| 0 | 1 |
| offset $=-2$ | 1 |
| offset $=-2$ | 0 |
| 0 | 1 |

## Stage 5

- From largest to smallest
- Armadillo - 1 -> 2
- Frog - 1 -> 7
- Bird - 5 -> 1
- Horse - 5 -> 9
- Iguana - $0->0$
- Elephant - $3->0$
- Cat - $4->0$
- Jaguar - 7 -> 5
- Dog - 8 -> 6
- Garage - 9 -> 8

| Intermediate <br> Table | Bit Array |
| :---: | :---: |
| offset $=0$ | 1 |
| reseed $=1$ | 1 |
| 0 | 1 |
| offset $=0$ | 1 |
| offset $=0$ | 1 |
| reseed = 1 | 1 |
| 0 | 1 |
| offset $=-2$ | 1 |
| offset $=-2$ | 1 |
| offset $=-1$ | 1 |

